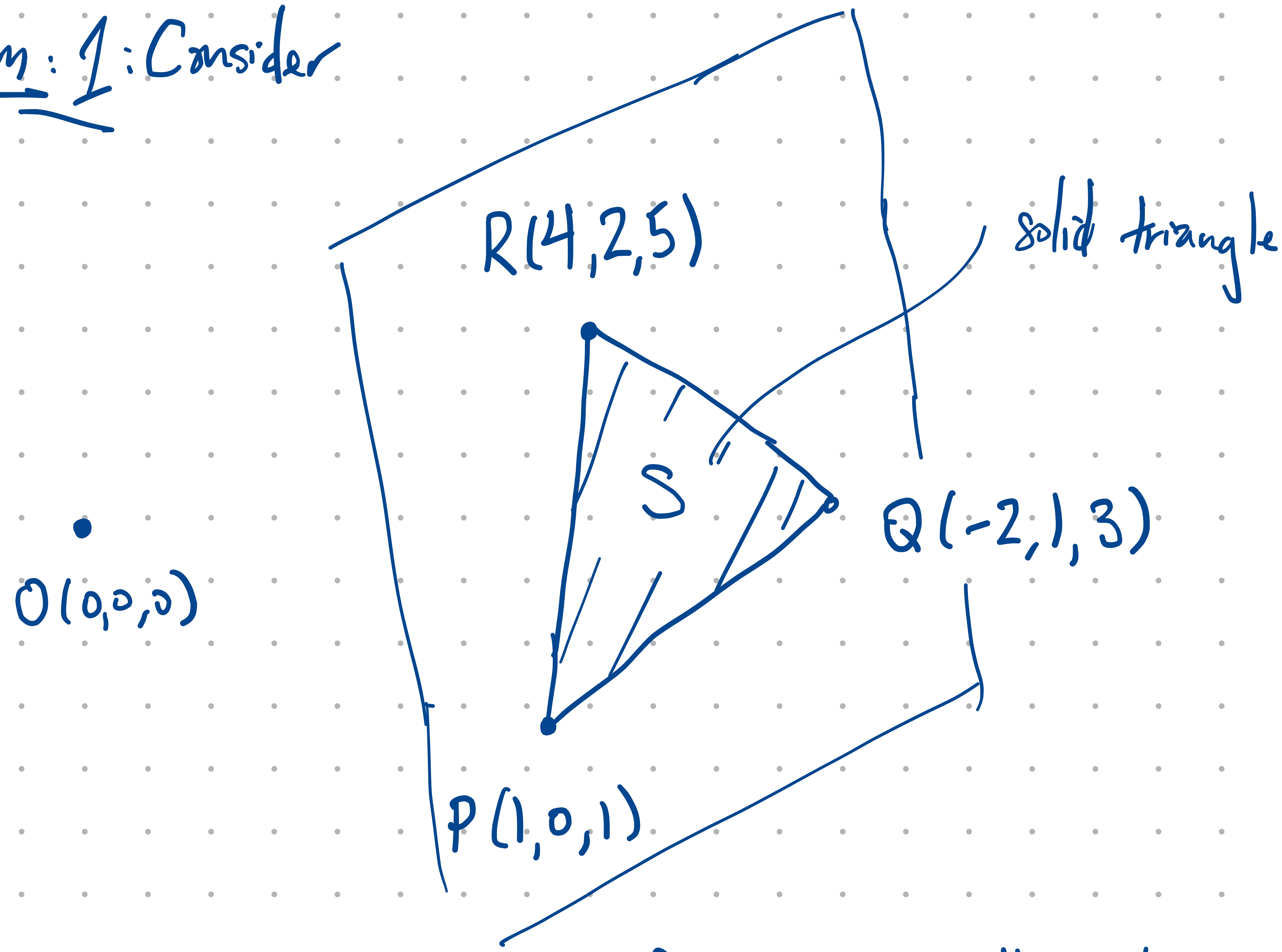
Problem 1: Consider

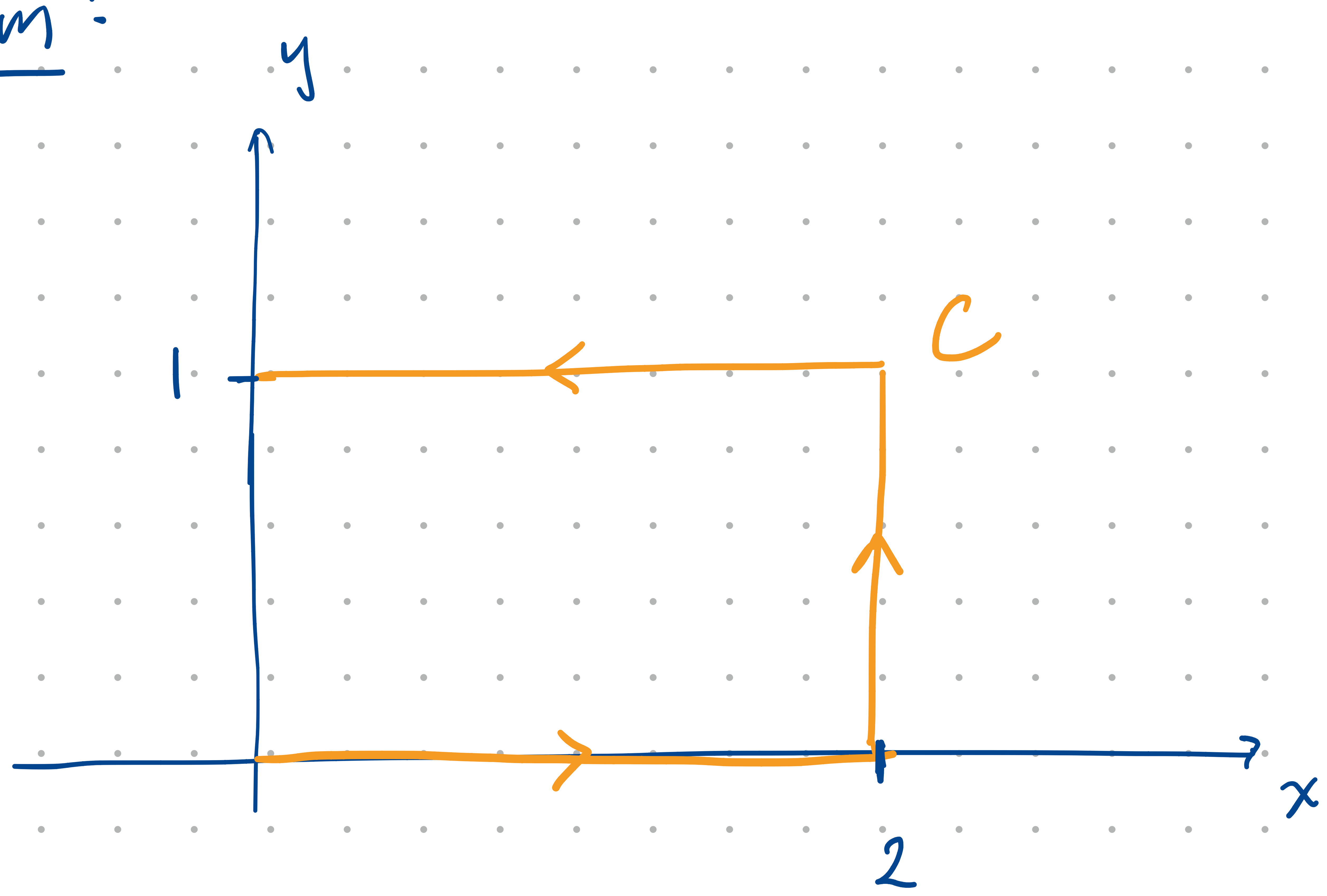


Compute the flux of $\langle x,y,z\rangle$ through S in the direction away from the origin.

a) Do this by first parametrizing S, and then setting up the integral directly

b) Can you figure out a way to solve this using the Divergence Thm?

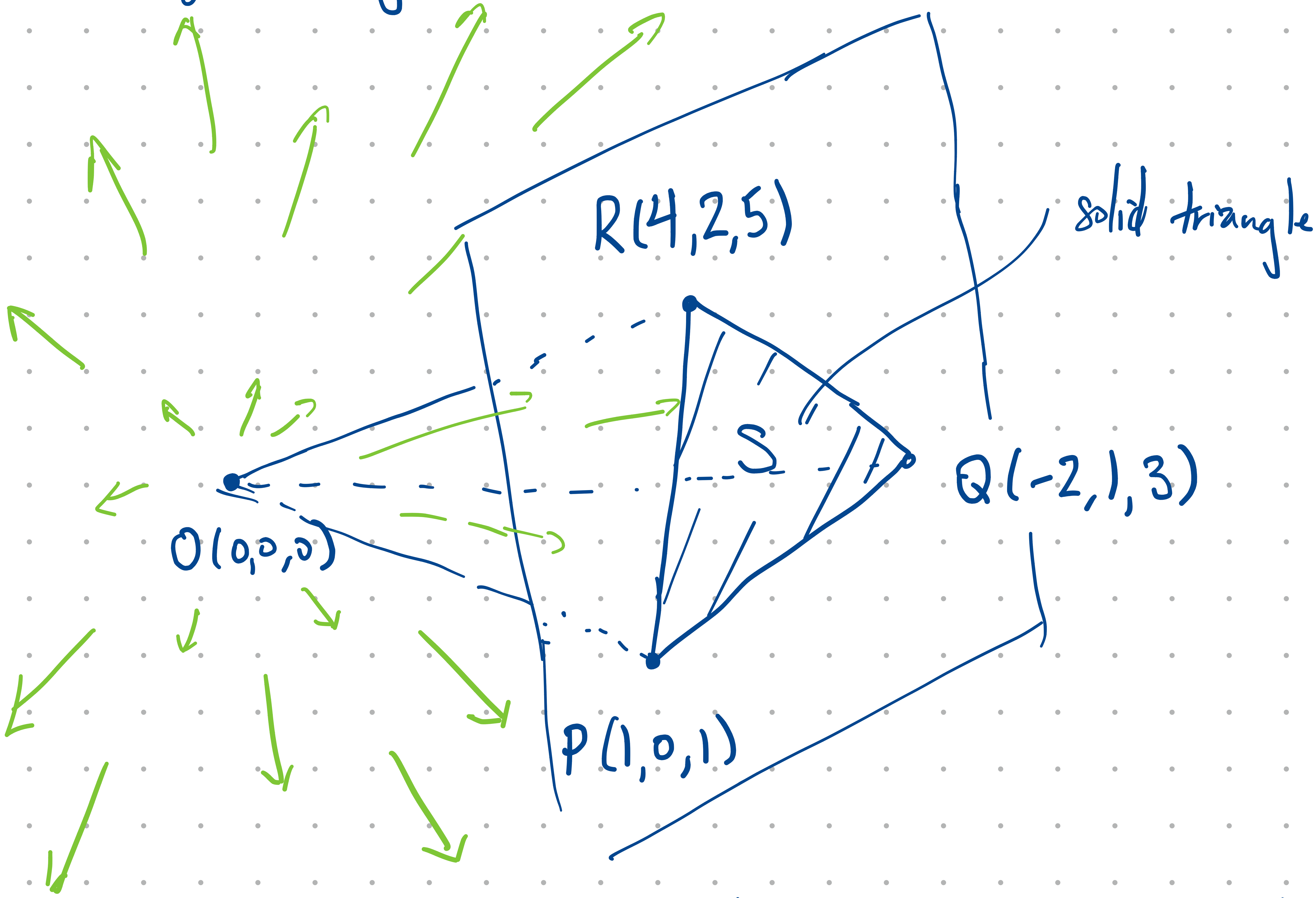
Problem:



Compute $\int_C \langle 2xy, x^2 + x \rangle \cdot d\vec{r}$ in as many different ways as you can think of.

e.g. direct computation, FTLI, Green's Thm...

A way of doing #1 w/ Divergence Thm:



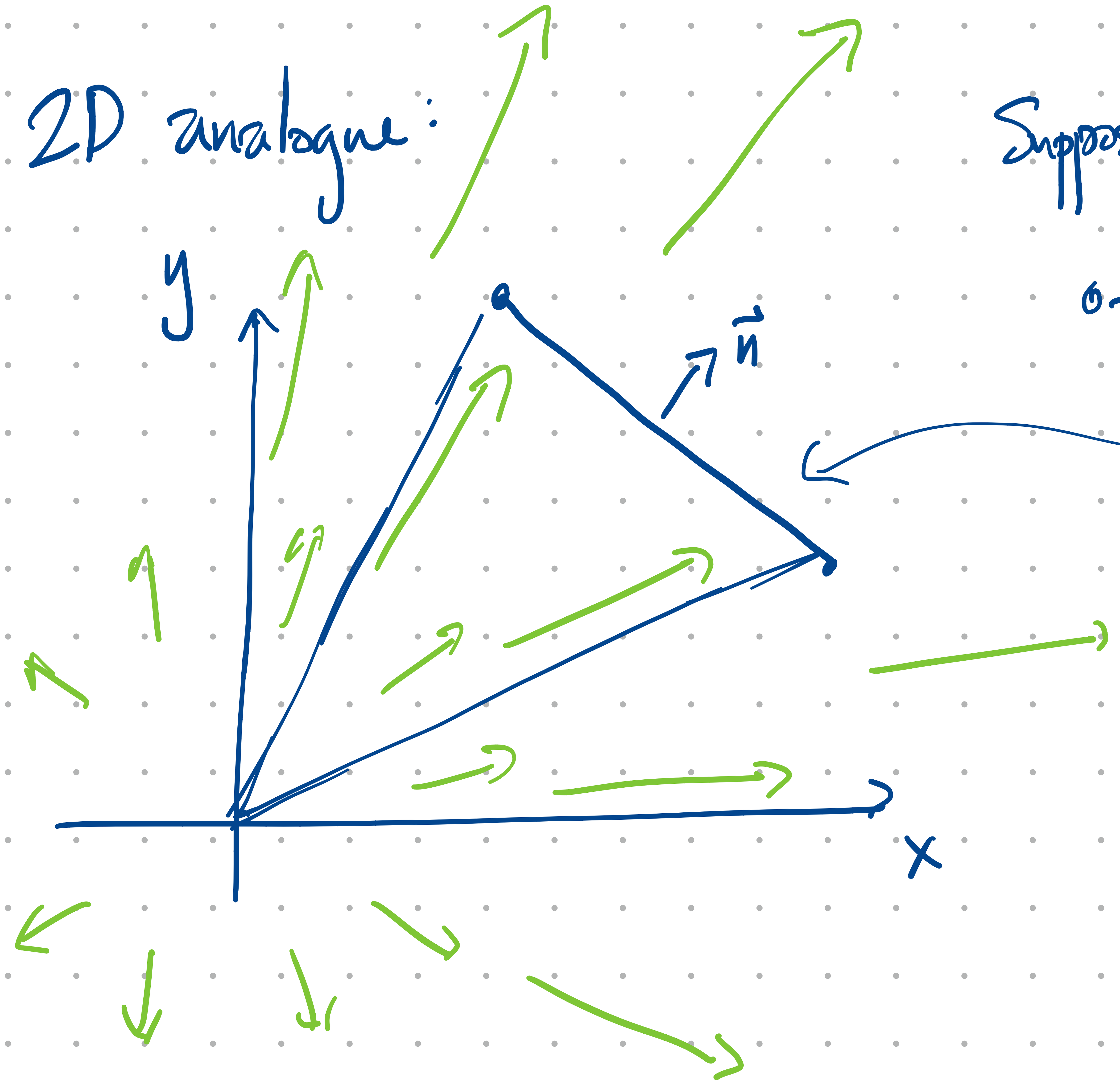
Consider the solid tetrahedron OPQR. Call it E.

Its boundary has four parts. The Divergence Thm says:

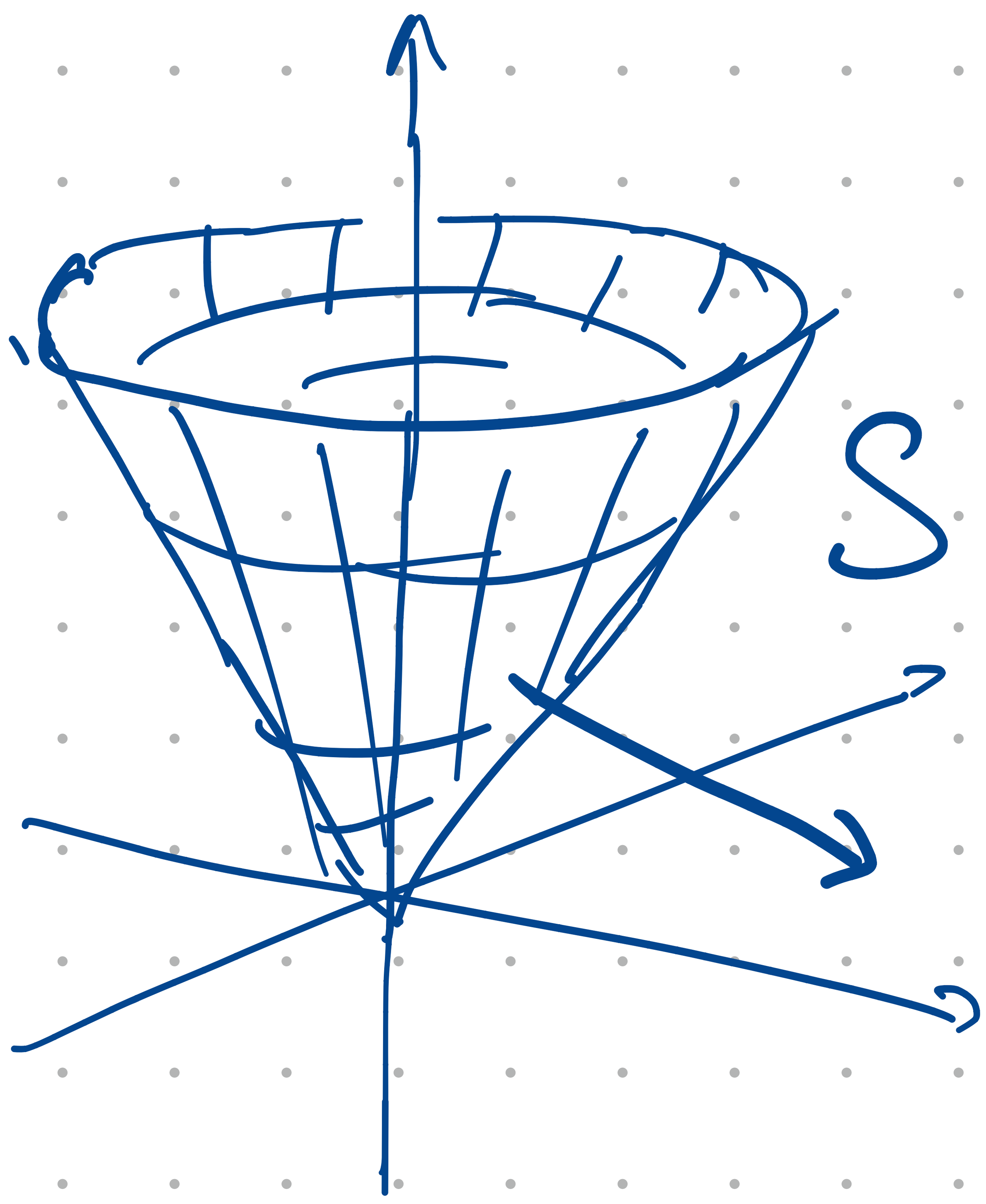
$$\iiint_E \left(\operatorname{div}\langle x,y,z\rangle\right) dV = \iint_{\substack{\text{Boundary} \\ \text{of } E}} \langle x,y,z\rangle \cdot d\vec{S}$$

$$= \iint_S \langle x,y,z\rangle \cdot d\vec{S} + \overbrace{\cancel{\iint_{\substack{\text{other} \\ \text{three faces}}} \langle x,y,z\rangle \cdot d\vec{S}}}^{0}$$

2D analogue:



Suppose I want flux of $\langle x,y \rangle$ through this



If asked to compute

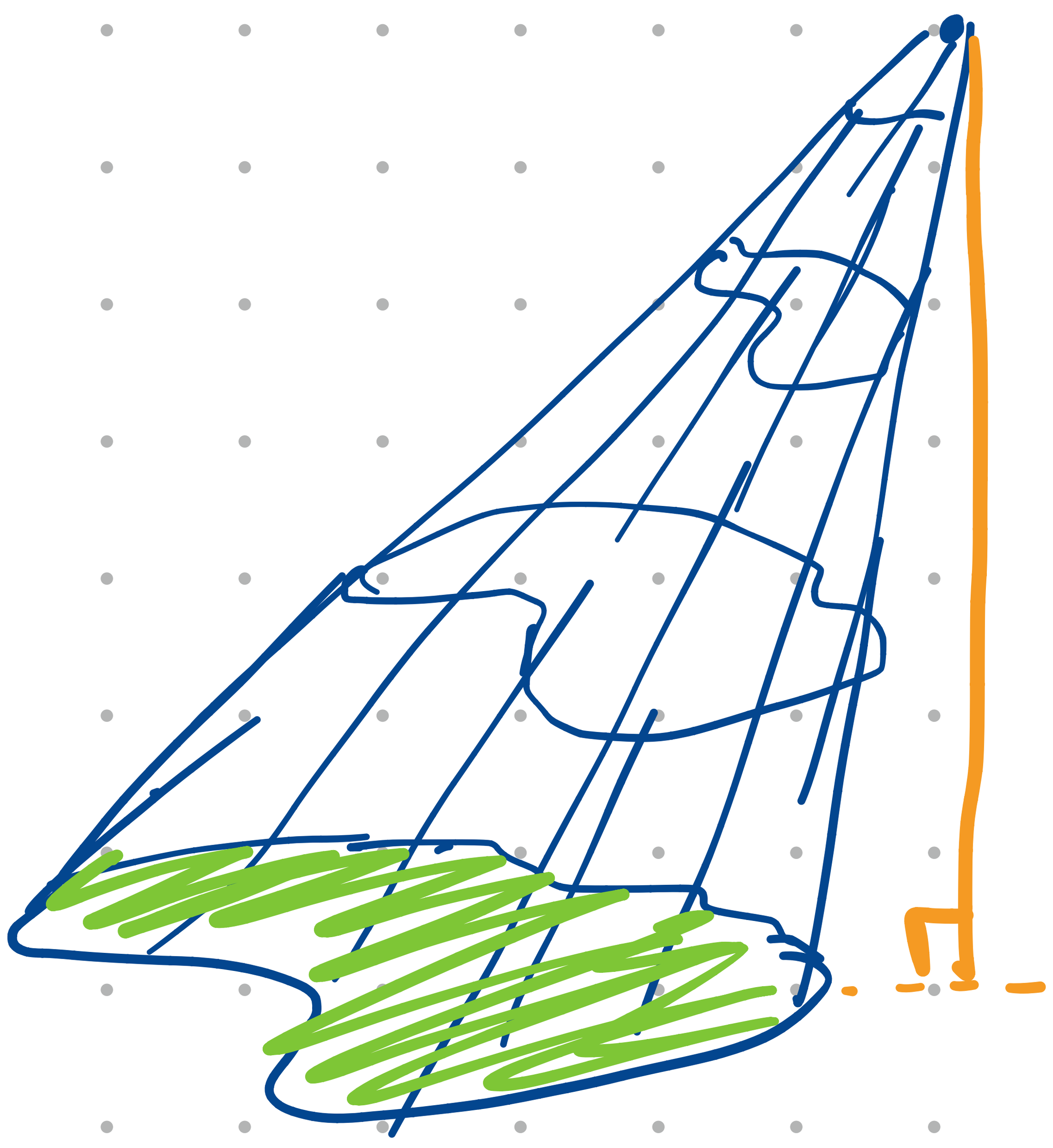
$$\iint_S \vec{F}\cdot d\vec{S}\,,$$

could compute directly, or

$$\iint_S \vec{F}\cdot d\vec{S} = \iiint \operatorname{div}\vec{F}\, dV - \iint \vec{F}\cdot d\vec{S}$$

So, need to compute

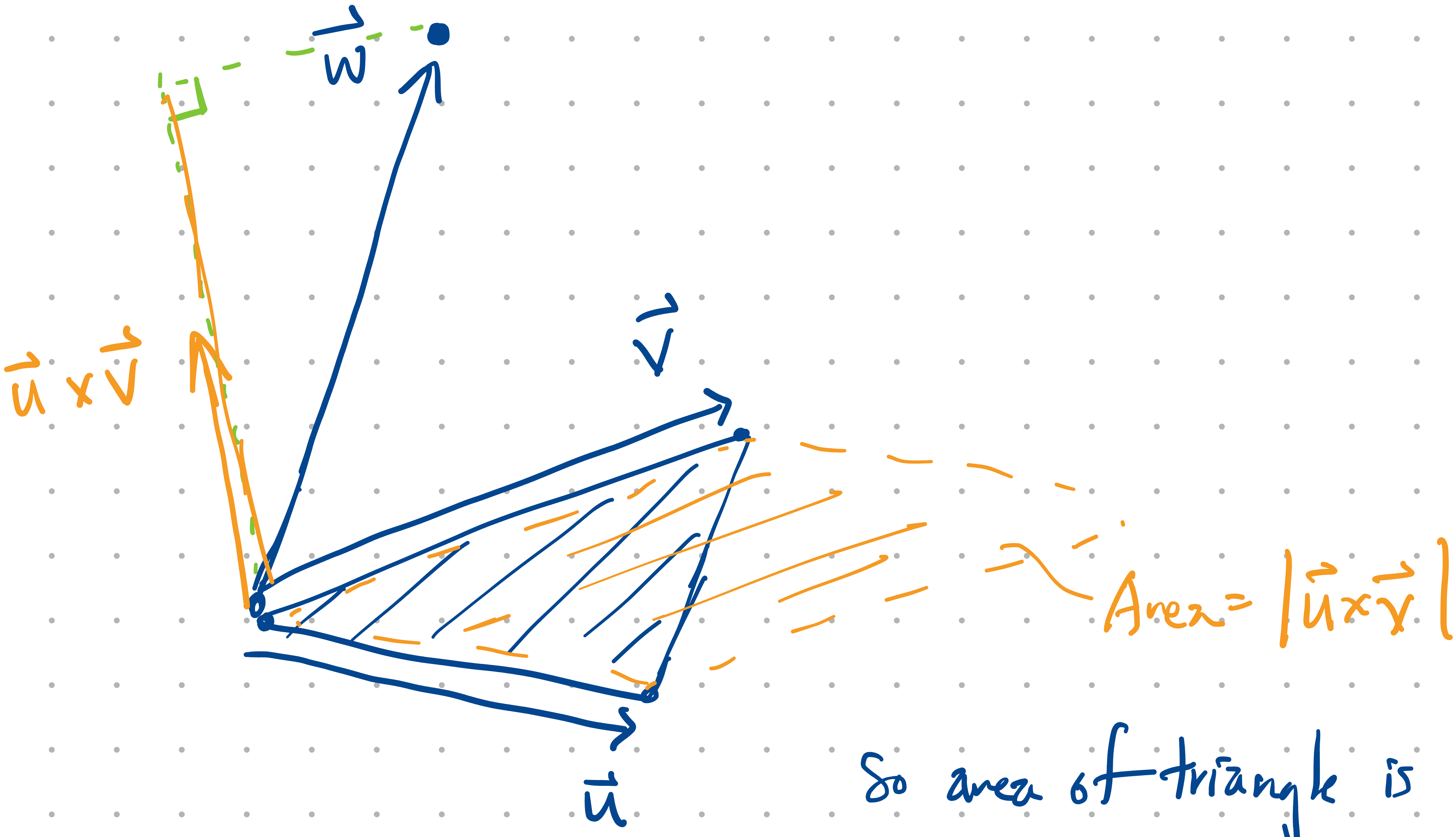
$$3 \cdot \mathrm{Vol}(\text{tetrahedron } OPQR).$$



General fact:

$$\mathrm{Vol} = \frac{1}{3}(\text{Area of Base})$$

· Height



So area of triangle is

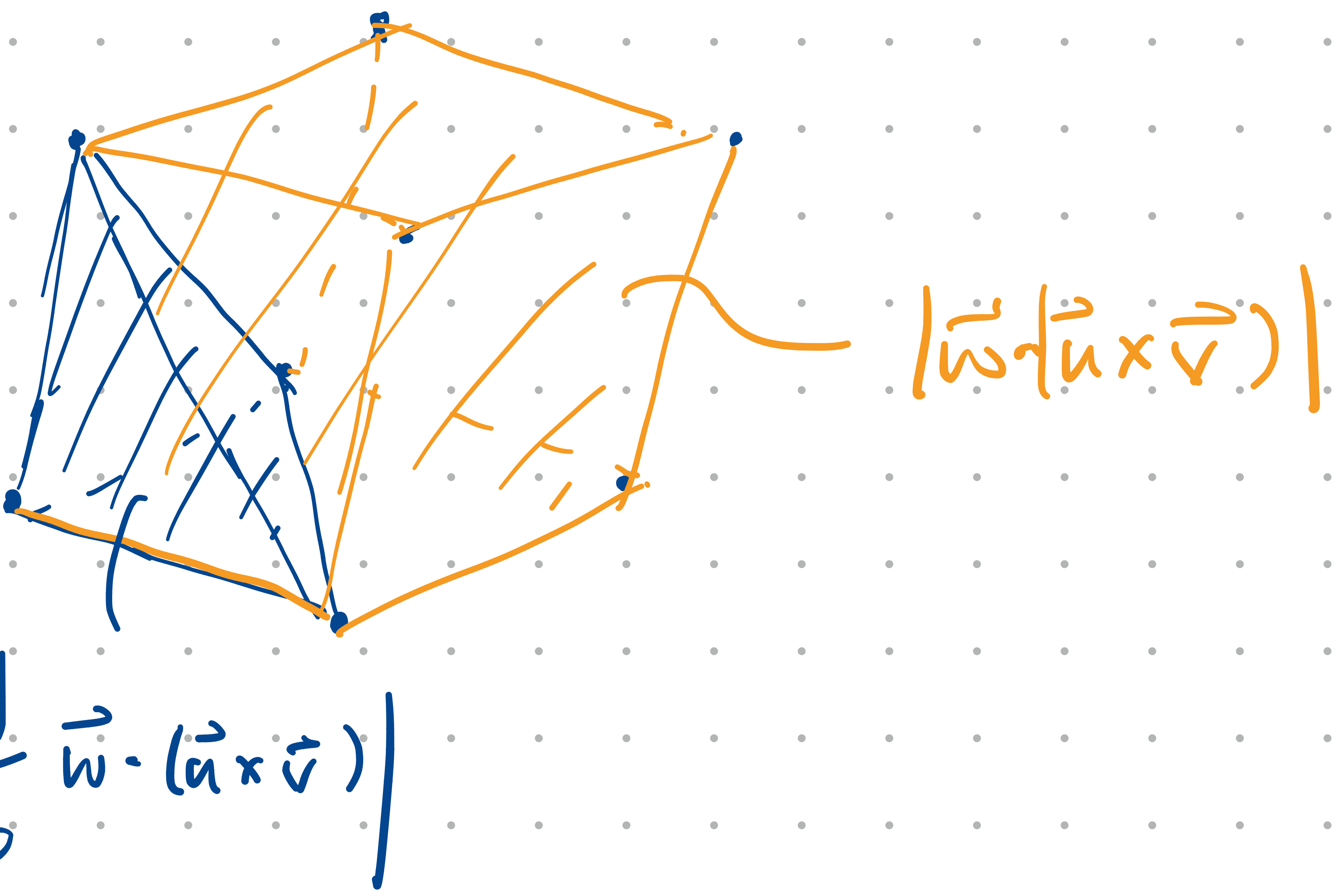
$$\frac{1}{2}|\vec{u} \times \vec{v}|.$$

So:

$$\text{Vol} = \frac{1}{3}\left(\frac{1}{2}\cancel{|\vec{u}\times\vec{v}|}\right)\left|\frac{\vec{w}\cdot(\vec{u}\times\vec{v})}{\cancel{|\vec{u}\times\vec{v}|}}\right|$$

$$= \left|\frac{1}{6}\,\vec{w}\cdot(\vec{u}\times\vec{v})\right|$$

Alternative interpretation:



Back to our problem:

$\vec{u} = \langle 1, 0, 1\rangle$

$\vec{v} = \langle 4, 2, 5\rangle$

$\vec{w} = \langle -2, 1, 3\rangle$

Compute:

$$\left|\frac{1}{6}\vec{w}\cdot(\vec{u}\times\vec{v})\right| = \frac{1}{6}\cdot 9$$

So final answer is $\boxed{9/2}$

$\vec{u} \cdot (\vec{v} \times \vec{w})$

$$\vec{v} \times \vec{w} = \det \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ — & v & — \\ — & w & — \end{bmatrix}$$

$$\|$$

If $\vec{u}'$ is the vector $a'\hat{i} + b'\hat{j} + c'\hat{k}$

and $\vec{u}$ is $\langle a, b, c \rangle$

$$\vec{u} \cdot \vec{u}' = a'a + b'b + c'c$$

i.e. replacing $\hat{i}, \hat{j}, \hat{k}$ w/ a, b, c.